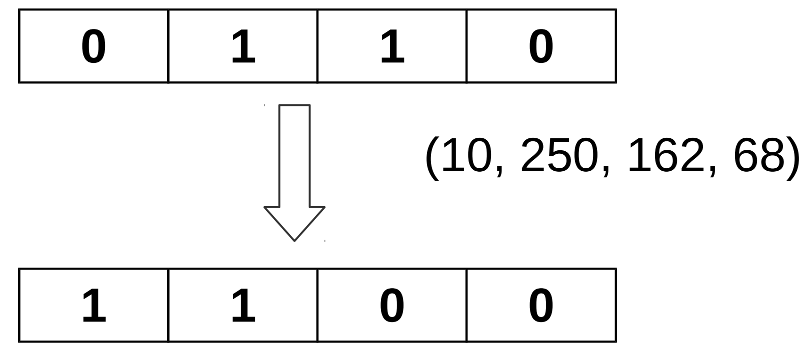
Cycle Structure of Non uniform Cellular Automata

Sukanya Mukherjee

Preliminaries

- 1 dimensional CA of finite size.
- 2 state, 3 neighborhood
- Null Boundary
- Linear/ Non-Linear CA
- Non-uniform CA



Rules and RMT Sequence

111 (7)	110 (6)	101 (5)	100 (4)	011 (3)	010 (2)	001 (1)	000 (0)	
d	d	d	d	1	0	1	0	10
1	1	1	1	1	0	1	0	250
1	0	1	0	0	0	1	0	162
d	1	d	0	d	1	d	0	68

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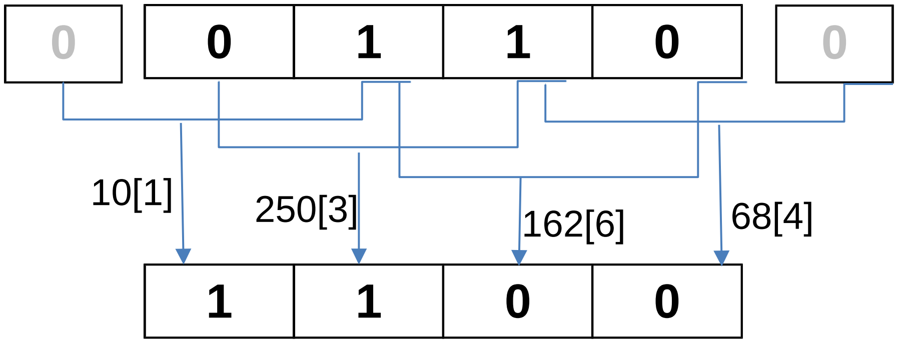

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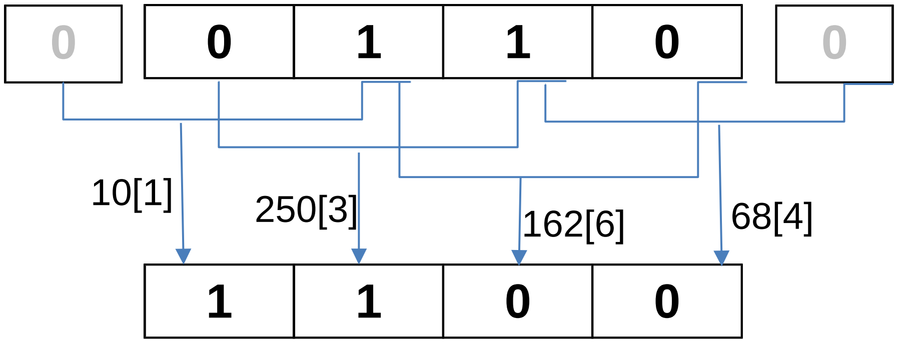

RMT at 0th position is one of 0,1,2,3
RMT at (n-1)th position is one of 2,4,6,0
RMT at ith position = (2 * RMT at (i-1)th position) mod 8
OR (2 * RMT at (i-1)th position +1) mod 8

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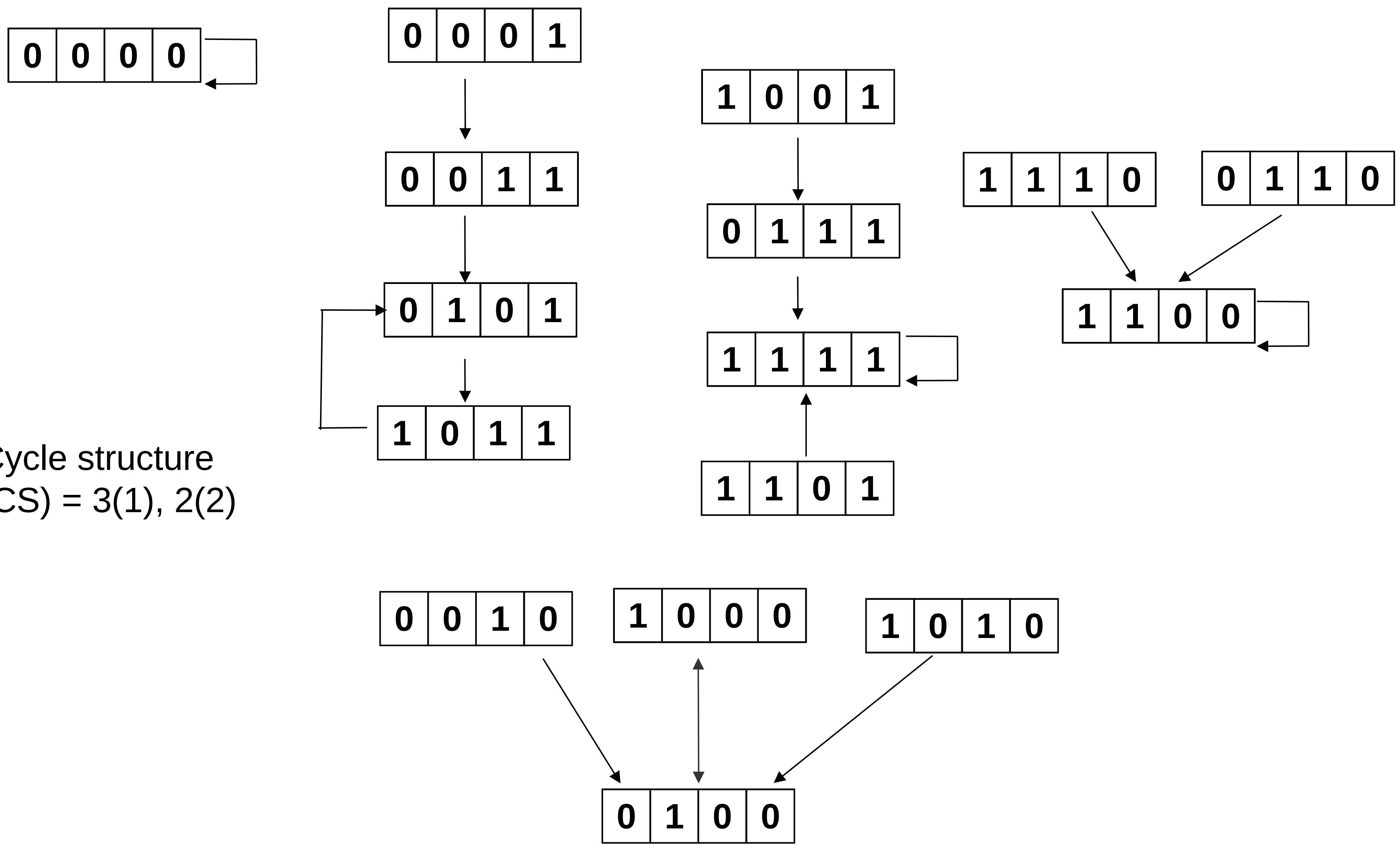

Cycle structure
(CS) = 3(1), 2(2)

Configuration transition diagram of CA (10, 250, 162, 68)

Motivation

- Application of cycle structure in the domain of Pseudo Random Pattern Generation, Cryptography.

- Some research had been done in Linear CA – using characteristic matrix, primitive polynomial.

- No work has been done on cycle structure in non-linear CA or non-uniform CA.

Reachability tree

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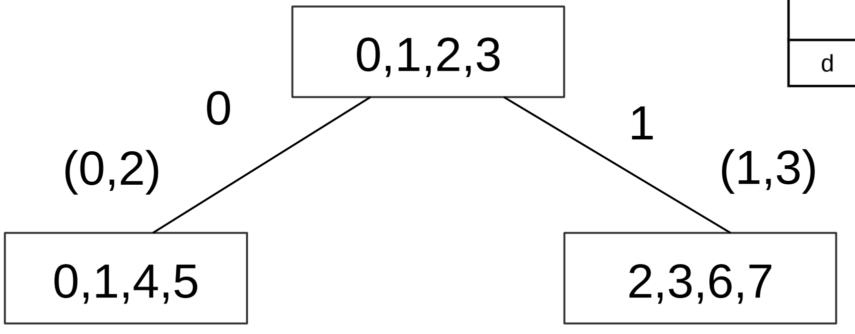

Reachability tree of CA (10, 250, 162, 68)

Reachability tree

11 1 (7)	11 0 (6)	10 1 (5)	10 0 (4)	01 1 (3)	01 0 (2)	00 1 (1)	00 0 (0)	
d	d	d	d	1	0	1	0	10
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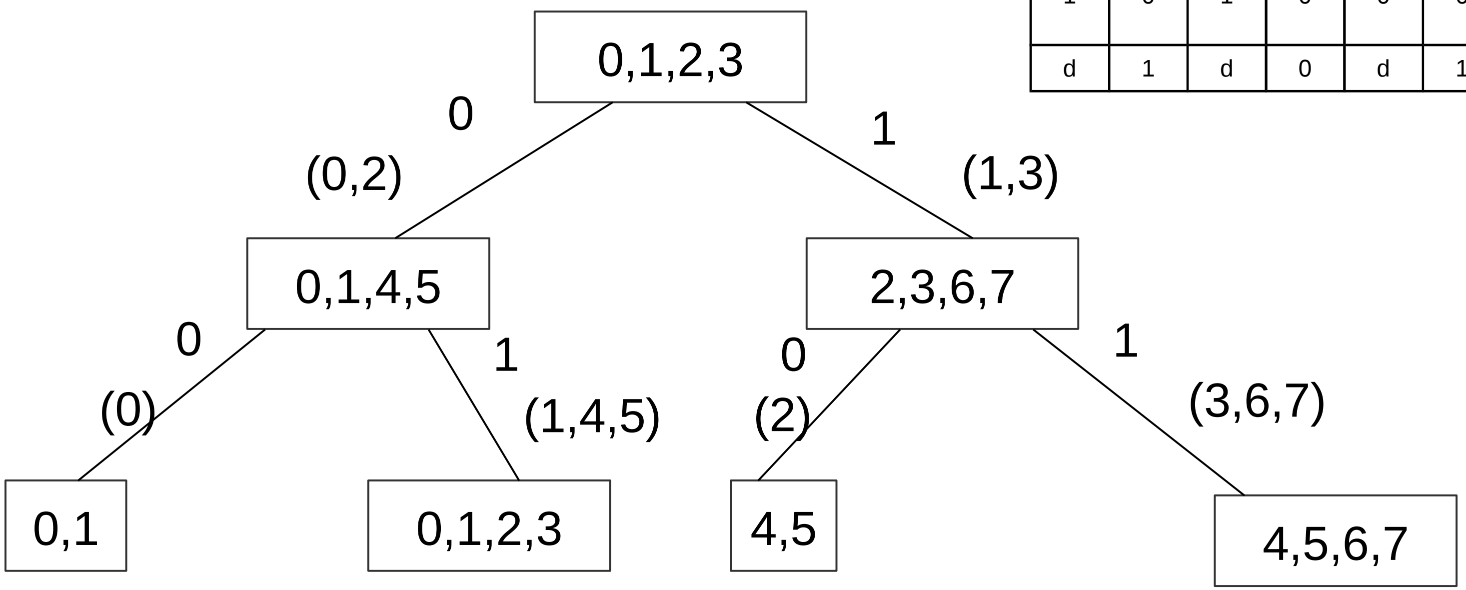

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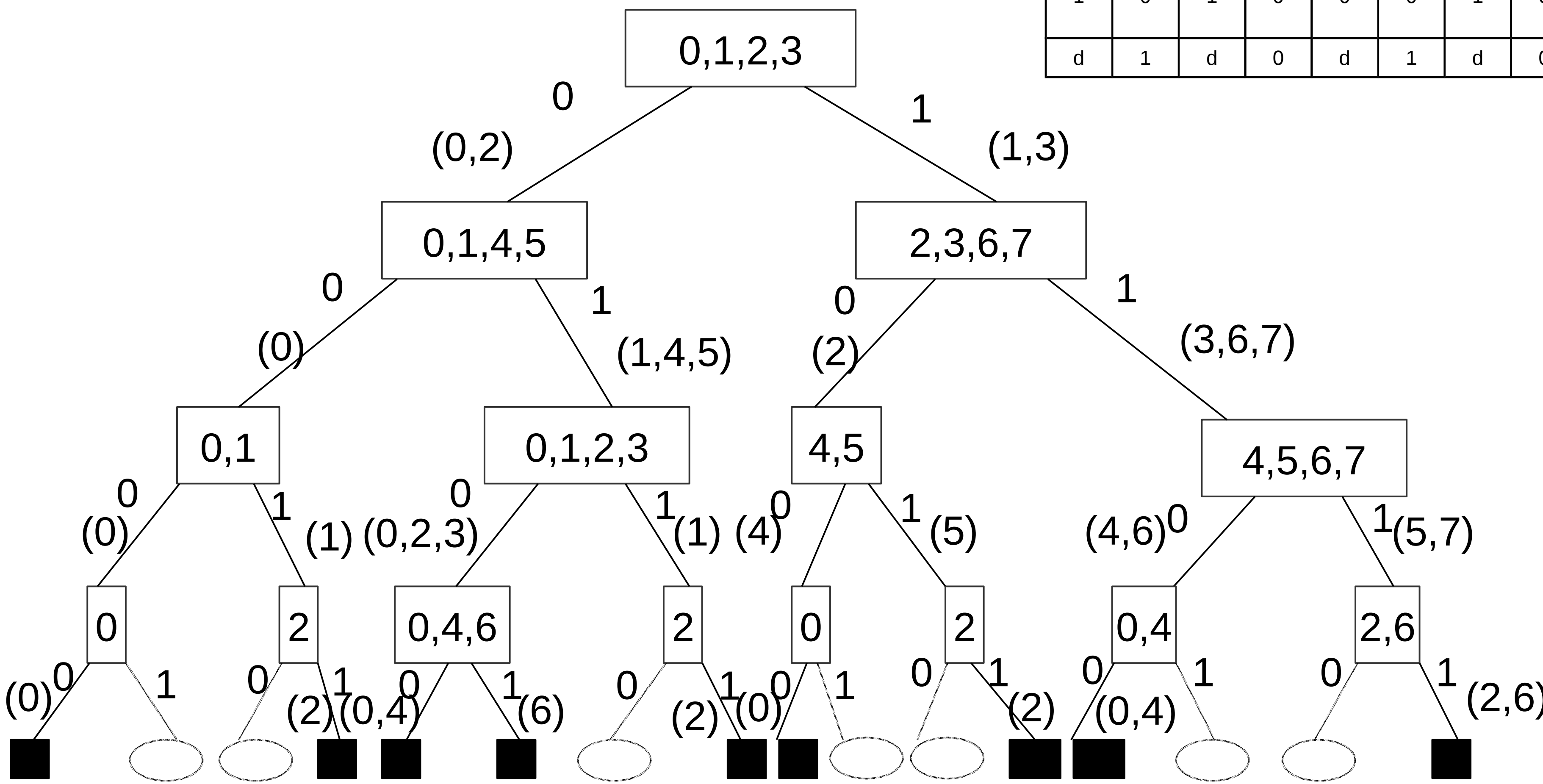

Reachability tree of CA (10, 250, 162, 68)

Configuration Transition in Reachability Tree

Green edges represent reachable configuration (0100)
Can be associated with RMT sequence (2400)
Configuration corresponding to RMT sequence (2400) is 1000.
Then 1000 is a predecessor of 0100
Link from 0100 to 1000.

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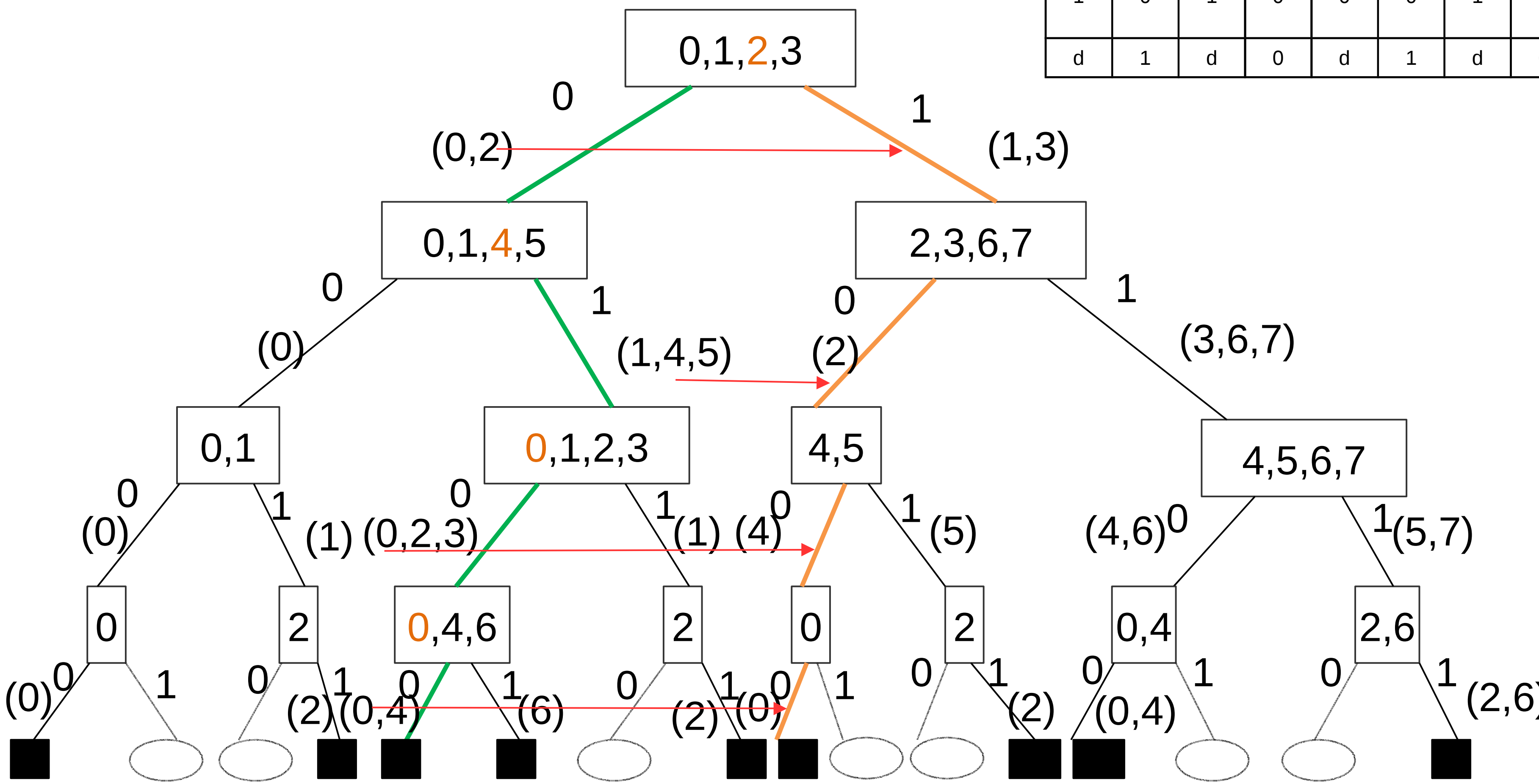
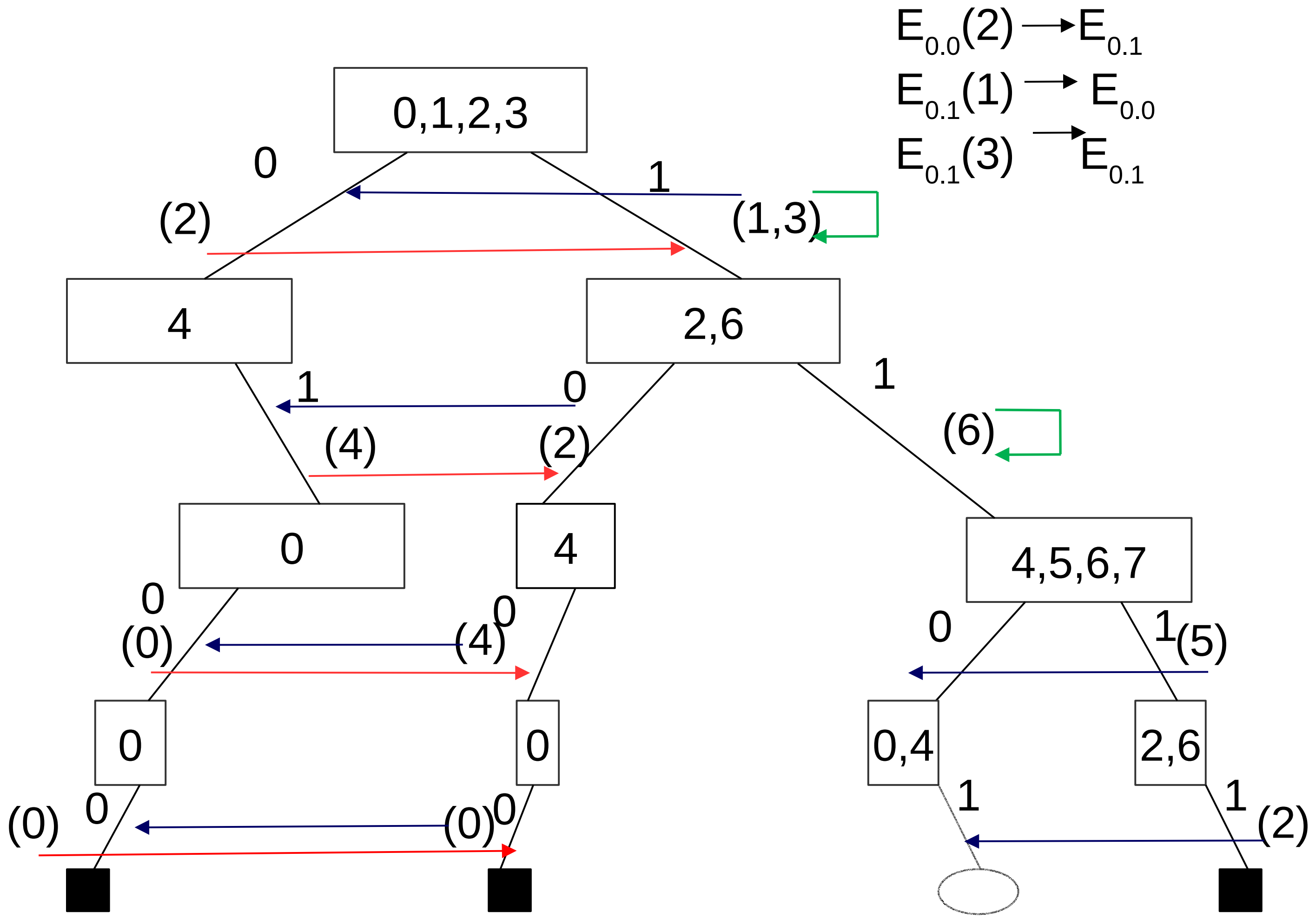


Reachability tree of CA (10, 250, 162, 68)

Links

Links can be forward, backward or self.



Identification of cycle structure

Cycles can be determined from the reachability tree using cross links and self links.

Crosslink : a sequence of forward and backward links forms cycle.

Crosslink of level i may be propagated to level i+1 or dissolved.

A crosslink of length m at level n-1 of reachability tree represents a cycle of length m of the given CA.

Identification of cycle structure

Removal of links which are not useful for cycle structure of the CA.

Identification of acyclic configuration – Finding of RMTs (links) not involved in any cross link or self link

⇩

Discard the links not involved in any loop

⇩

Keep only those links which form crosslinks and self links at level n-1

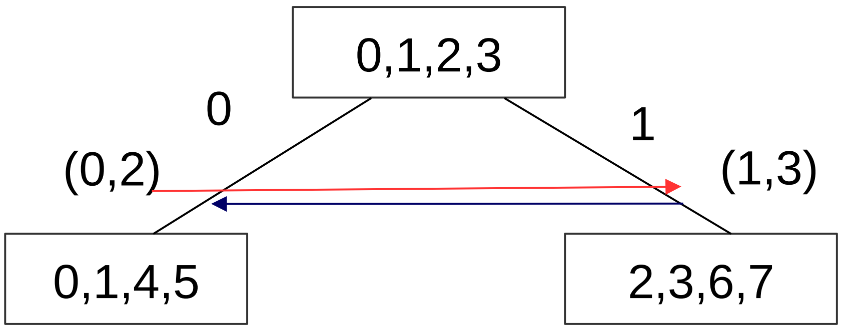
⇩

At level n-1, count the crosslinks along with length. Count the self links also.

Determining of Cycles

$lc_0^I=4$; $ec_0^I=2$;

$lc_0^F=4$; $ec_0^F=2$;

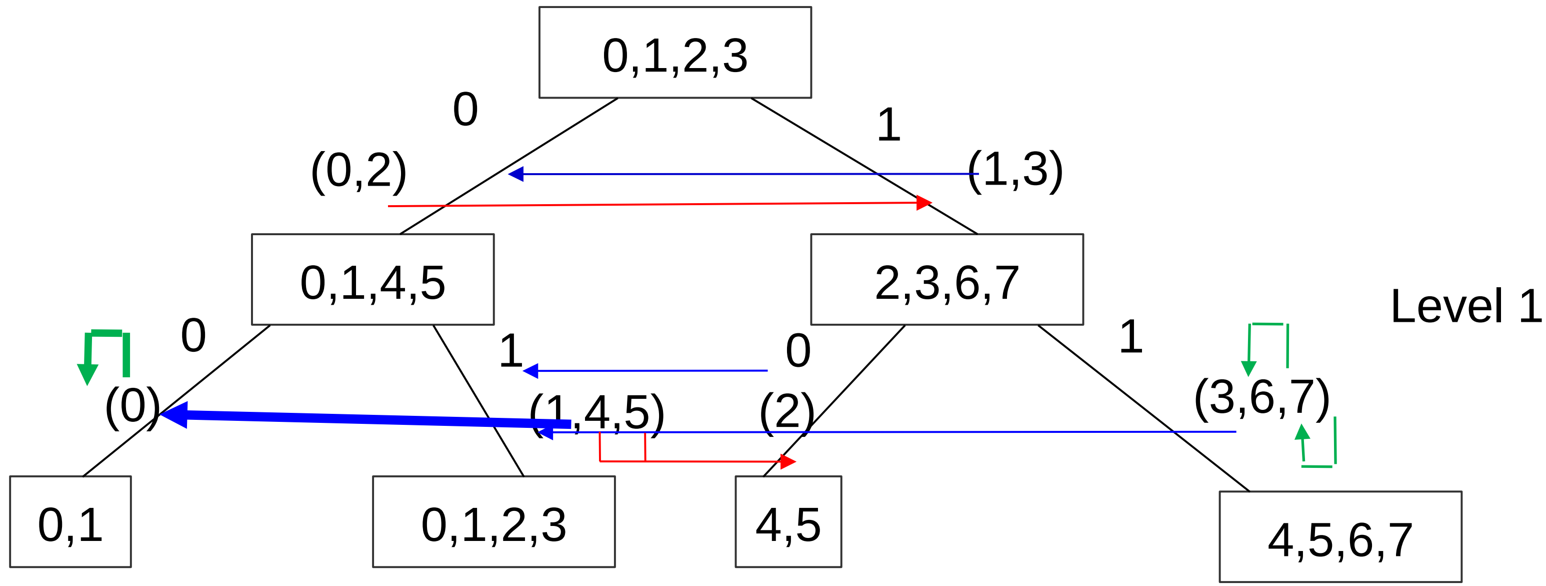


Level 0

Determining of Cycles

The sole RMT for an edge forms a self link and there is a link to the edge for sibling RMT □ the latter link can be deleted.

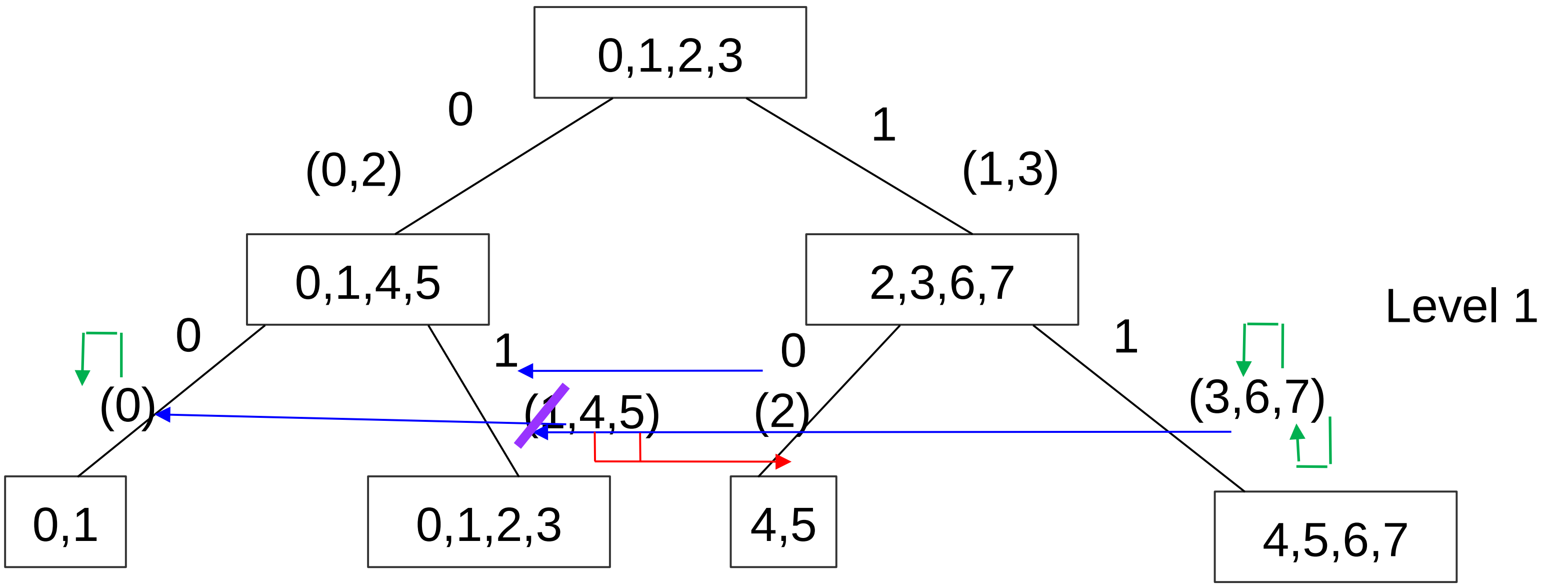
$lc_1^l=8; ec_1^l=4;$



Determining of Cycles

$lc_1^I = 8; \; ec_1^I = 4;$

$lc_1^F = 7; \; ec_1^F = 4;$

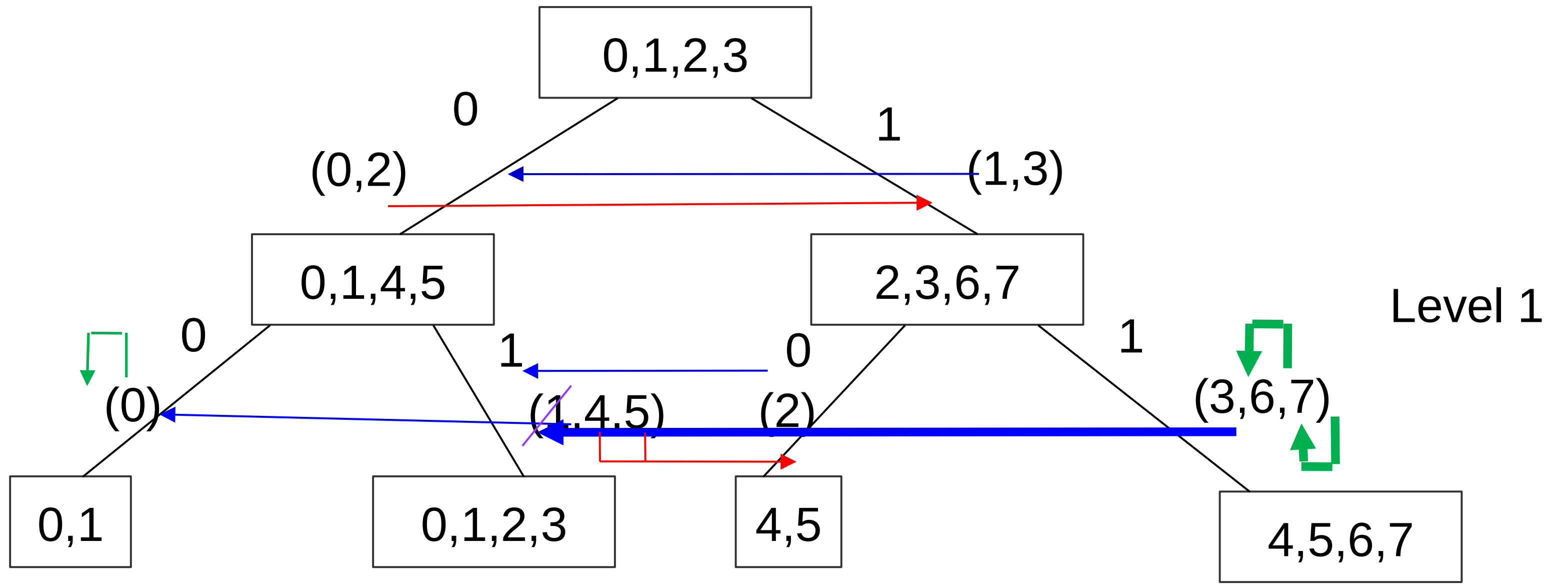


Determining of Cycles

Self links for two sibling RMTs on same edge All remaining links from that edge for remaining RMTs should be deleted.

$lc_1^I=8$; $ec_1^I=4$;

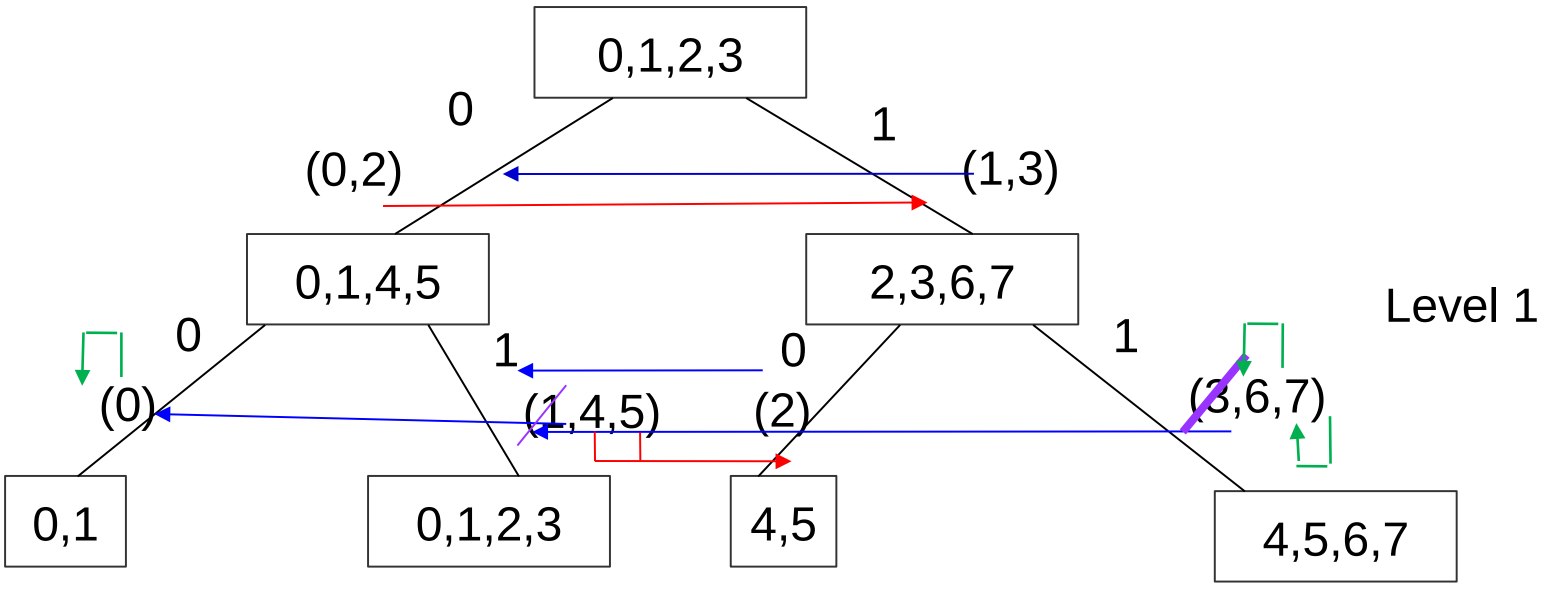
$lc_1^F=6$; $ec_1^F=4$;



Determining of Cycles

$lc_1^I = 8; \; ec_1^I = 4;$

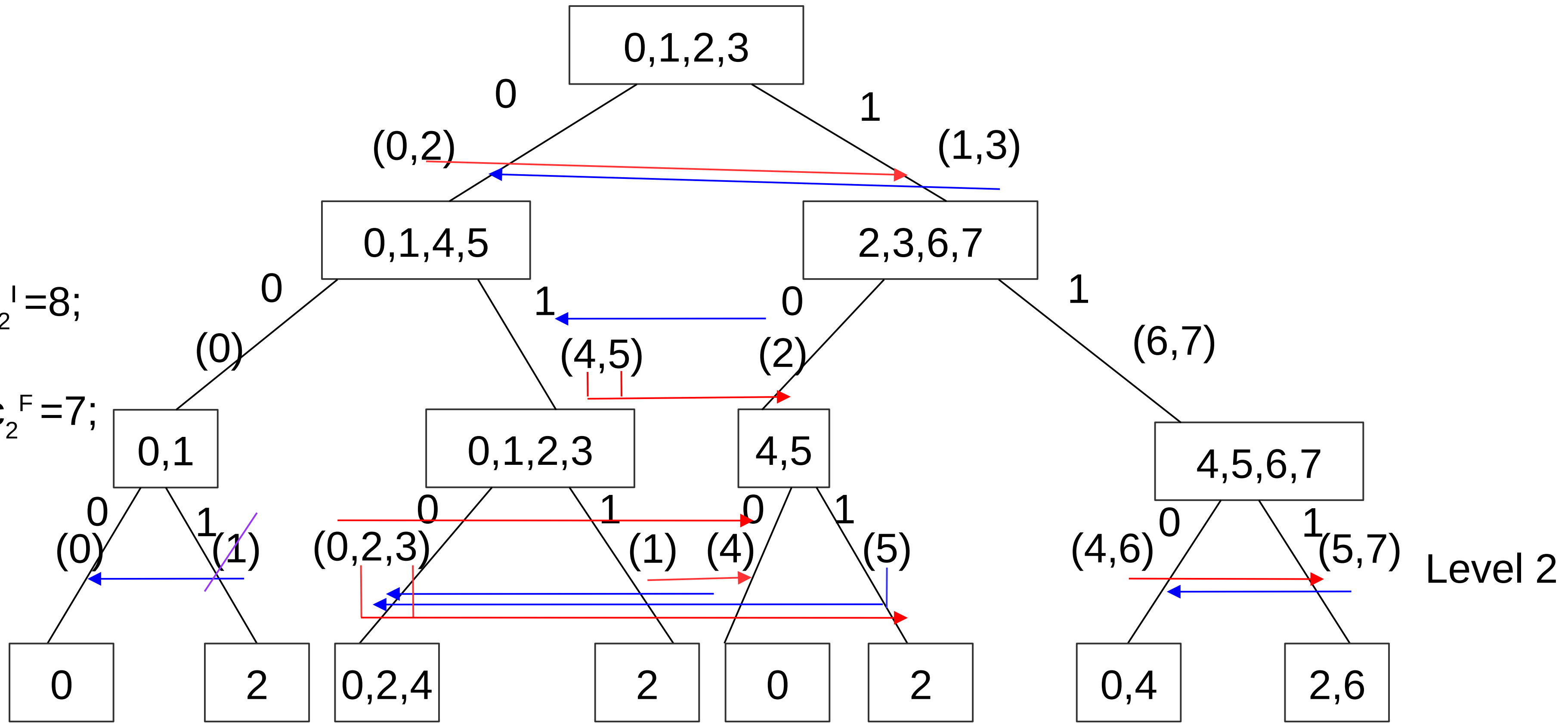
$lc_1^F = 6; \; ec_1^F = 4;$



Determining of Cycles

$lc_2^I = 12;\ ec_2^I = 8;$

$lc_2^F = 11;\ ec_2^F = 7;$



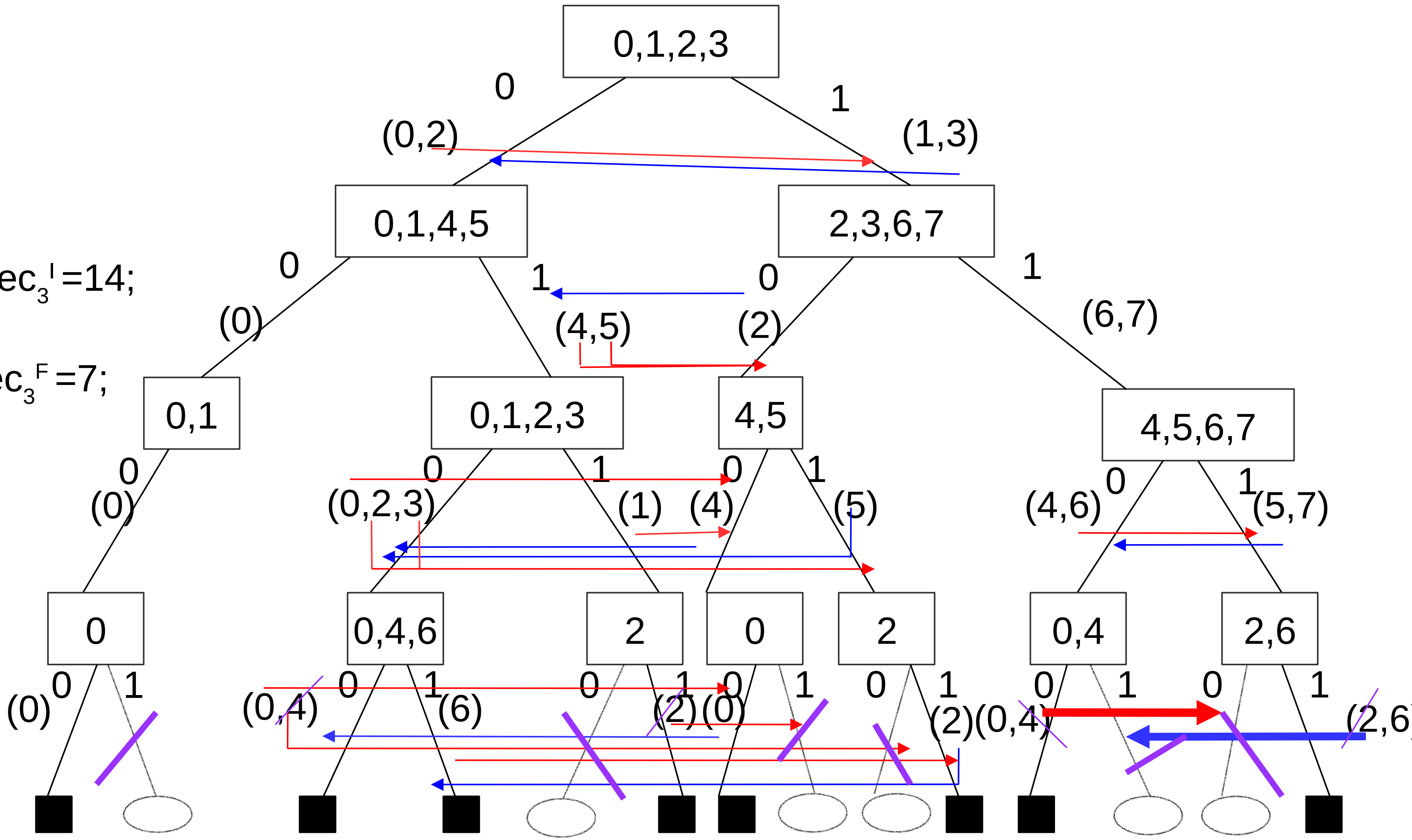
Determining of Cycles

$E_{i,j}(r) \longrightarrow E_{i,j'}$ and $E_{i,j'}$ is non-reachable edge

Discard the link $E_{i,j}(r) \longrightarrow E_{i,j'}$.

$lc_3^I=11; ec_3^I=14;$

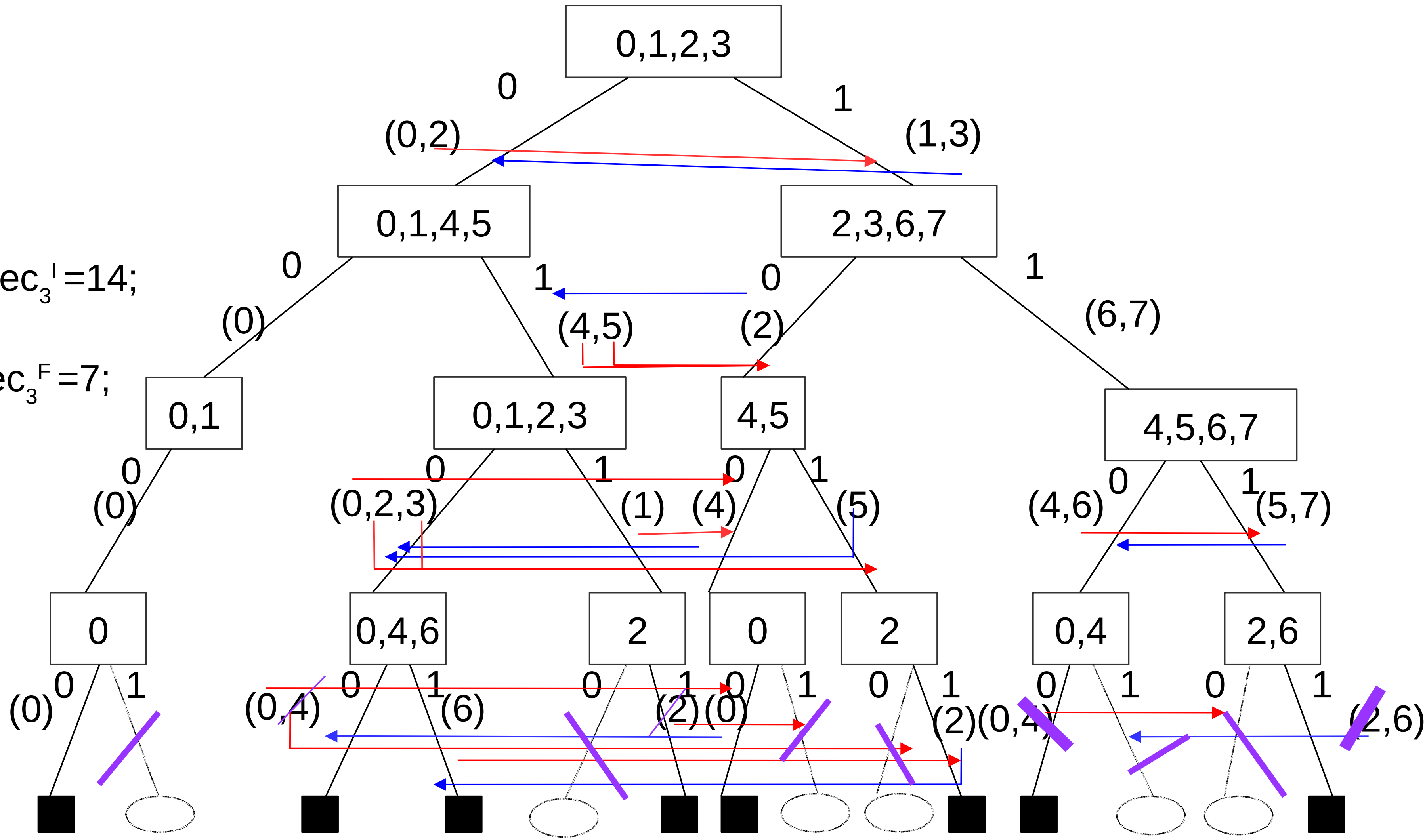
$lc_3^F=7; ec_3^F=7;$



Determining of Cycles

$lc_3^I=11; ec_3^I=14;$

$lc_3^F=7; ec_3^F=7;$



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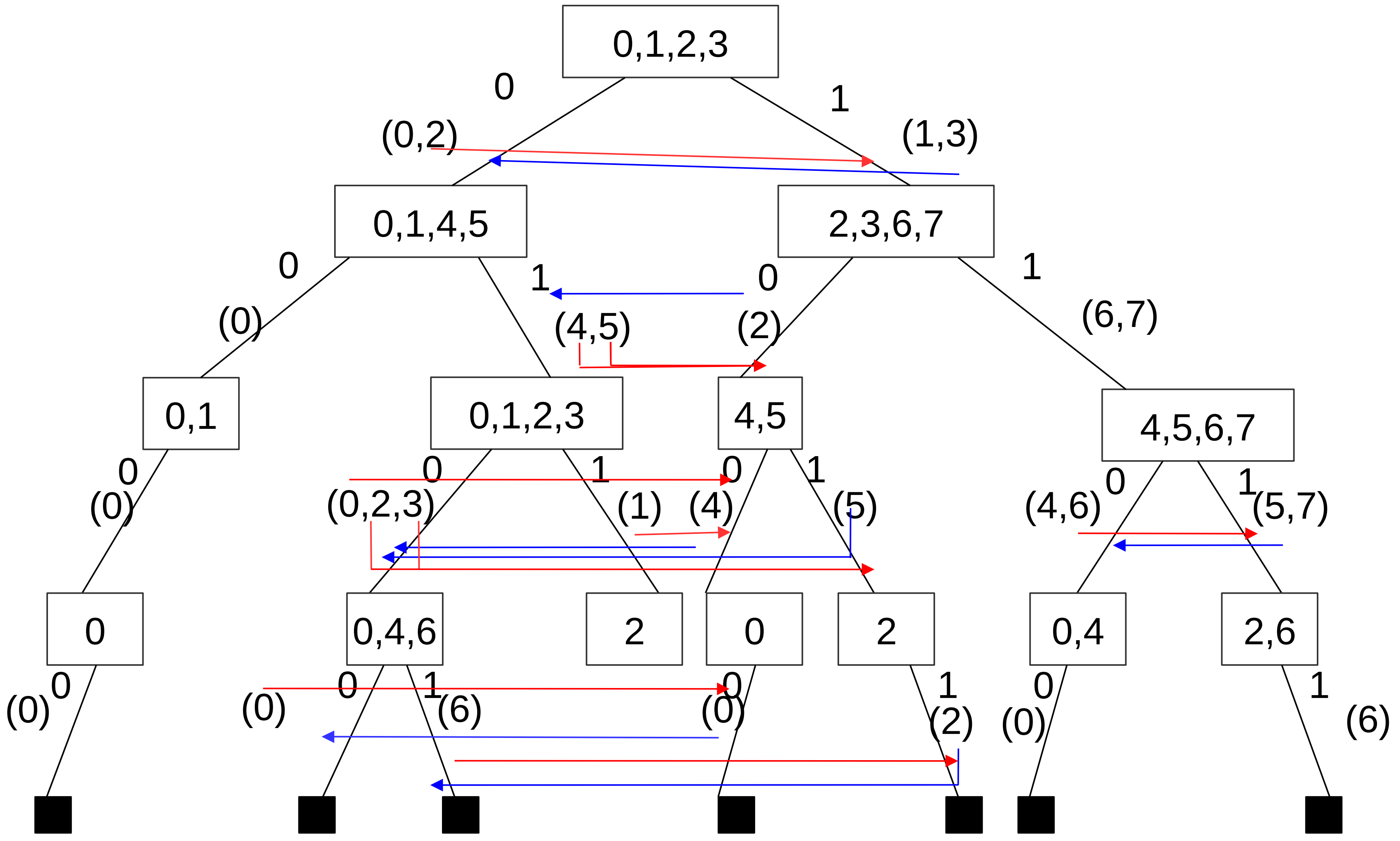

Experimental Results

CA size	Rule Vector	
10	$(90,150)^5$	1(1), 1(1023)
12	$(30^4,45^2)^2$	1(1),2(3),1(4),1(8)
18	$((90,150)^2,240^2)^3$	1(1),2(6),1(3),20(12),160(24)
21	$(33,64,29,60,8,10,25)^3$	162(6)
24	$(4^5,90^3)^3$	1597(1)
27	$(170,64,96,192,102,45,86,13,240)^3$	1(1), 168(3)
28	$(102,195,8,10)^7$	1(1)
35	$(4,186,15,240,80)^7$	4096(4)
42	$(8,45,30,120,64,48)^7$	1(1),5461(3)
55	$(1,21,35,95,48)^{11}$	1024(2)
100	$(8^{50},48^{50})$	1(1)
500	$(90^2,240^2)^{125}$	1(1)

Conclusion and Future Work

- Studied the problem of determining cycle structure of a finite one dimensional non-uniform CA.
- Novel scheme where only the relevant configurations are kept in the reachability tree.
  - reduces space and time complexity.
  - performs efficiently mainly for irreversible CAs (experimentally verified)
- Open problems
  - reduce the space complexity for reversible CAs.
  - study the reverse problem, i.e., constructing a CA which generates cycles of given length.

THANK YOU